

On α' corrections in $\mathcal{N} = 1$ F-theory compactifications

Thomas W. Grimm, Raffaele Savelli and Matthias Weißenbacher
Max-Planck-Institut für Physik, Munich, Germany

We consider $\mathcal{N} = 1$ F-theory and Type IIB orientifold compactifications and derive new α' corrections to the four-dimensional effective action. They originate from higher derivative corrections to eleven-dimensional supergravity and survive the M-theory to F-theory limit. We find a correction to the Kähler potential depending on the volume of a non-trivial intersection curve of seven-branes. At weak string coupling this correction arises from the intersection of D7-branes with the O7-plane. We also analyze a four-dimensional higher curvature correction.

I. INTRODUCTION

F-theory is a formulation of Type IIB string theory with seven-branes at varying string coupling [1]. It captures string coupling dependent corrections in the geometry of an elliptically fibered higher-dimensional manifold. The general effective actions of F-theory compactifications have been studied using the duality with M-theory [2, 3]. M-theory is accessed through its long wave-length limit provided by eleven-dimensional supergravity. This implies that the F-theory effective actions are perturbative in the string tension α' . Starting with the two-derivative supergravity action one derives the classical F-theory effective action. Studying α' corrections to this action is of crucial importance for many questions both at the conceptual and phenomenological level. In particular, a central task is the analysis of moduli stabilization in four-dimensional (4d), $\mathcal{N} = 1$ F-theory compactifications [2].

In this work we study a set of α' corrections to 4d, $\mathcal{N} = 1$ F-theory effective actions arising from known higher-derivative terms in the 11d supergravity action. More precisely, we find all corrections induced by a classical Kaluza-Klein reduction of the purely gravitational M-theory R^4 -terms investigated in [4–9] on elliptically fibered Calabi-Yau fourfolds. We implement the F-theory limit decompactifying the 3d M-theory reduction to four space-time dimensions and interpret the resulting corrections in F-theory. Two α'^2 corrections are shown to survive the limit. We find a correction to the volume of the Calabi-Yau fourfold base appearing in the Kähler potential and an R^2 -term in the 4d effective action. Both only depend on the Kähler moduli of the $\mathcal{N} = 1$ reduction. The presence of a volume correction in the M-theory reduction on Calabi-Yau fourfolds has already been stressed in [10, 11]. Moreover, it was found in [12] that a general M-theory reduction on a Calabi-Yau fourfold also includes a warp factor. In this work we will neglect warping effects. There is no warp factor in six dimensions and we comment on the α'^2 corrections in Calabi-Yau threefold reductions of F-theory.

To give an independent interpretation of these two α'^2 corrections we take the Type IIB weak string coupling limit [13]. The F-theory volume correction is proportional to the volume of the intersection curve of the D7-branes with the O7-plane. A simple counting of powers

of the string coupling suggests that this correction arises from tree-level string amplitudes involving oriented open strings with the topology of a disk, and non-orientable closed strings with the topology of a projective plane. This interpretation is at odds with the expectation that such a correction arises at open-string one-loop level [14]. It would be crucial to clarify this point and to obtain an independent string derivation. Within our approach one needs to check if there are further corrections in a fully backreacted M-theory reduction lifted to F-theory that have the same structure as the volume correction found here. The 4d higher curvature correction is matched with a higher curvature modification of the Dirac-Born-Infeld actions of D7-branes and O7-planes derived in [15]. Different α' corrections to F-theory effective actions and their weak coupling interpretations have been found in [16, 17]. A class of α'^2 corrections in the heterotic string has been discussed recently in [18].

II. M-THEORY REDUCTION

Our starting point is the long wave-length limit of M-theory given by 11d supergravity. In particular, we focus on a well-known higher derivative correction to the Einstein-Hilbert term of the form [4–9]

$$S_{(11)} \supset \frac{1}{(2\pi)^8 l_M^9} \int *_{11} \mathbf{1} \{ R_{\text{sc}}^{(11)} + \frac{\pi^2 l_M^6}{3^2 2^{11}} \mathcal{J}_0 \}, \quad (1)$$

where $*_D \mathbf{1} = d^D X \sqrt{-G^{(D)}}$ is the D -dimensional volume element, $R_{\text{sc}}^{(D)}$ is the D -dimensional Ricci scalar, and l_M is the 11d Planck length. The correction is given by a Lorentz invariant combination of four powers of the Riemann tensor $R^{(11)}$ of the schematic form

$$\mathcal{J}_0 = t_8 t_8 (R^{(11)})^4 - \frac{1}{4!} \epsilon_{11} \epsilon_{11} (R^{(11)})^4, \quad (2)$$

where the precise form of the individual terms is given in (A5) and (A6). In this work we follow the conventions of [9]. In these conventions the metric is dimensionless and only the space-time coordinates have dimensions.

If we now compactify this theory on a Calabi-Yau fourfold Y_4 , the resulting 3d effective action will include the curvature terms (before Weyl rescaling) of the form

$$S_{(3)} \supset \frac{1}{(2\pi)^8 l_M} \int *_3 \mathbf{1} \{ \tilde{\mathcal{V}}_4 R_{\text{sc}}^{(3)} + l_M^2 \tilde{\mathcal{V}}_2 |\mathcal{R}^{(3)}|^2 \}, \quad (3)$$

where $\mathcal{R}^{(D)} = \frac{1}{2}R_{\mu\nu}^{(D)} dx^\mu \wedge dx^\nu$ is the curvature two-form in D dimensions, and one has

$$\begin{aligned} |\mathcal{R}^{(D)}|^2 *_D \mathbf{1} &= \text{Tr}(\mathcal{R}^{(D)} \wedge *_D \mathcal{R}^{(D)}) \\ &= -\frac{1}{8} R_{\mu\nu\lambda\rho}^{(D)} R^{(D)\mu\nu\lambda\rho} *_D \mathbf{1}. \end{aligned} \quad (4)$$

The volumes appearing in (3) take the form

$$\tilde{\mathcal{V}}_4 = \frac{1}{4!} \int_{Y_4} J^4 + \frac{\pi^2}{24} \int_{Y_4} c_3(Y_4) \wedge J, \quad (5)$$

$$\tilde{\mathcal{V}}_2 = \frac{\pi^2}{24} \int_{Y_4} c_2(Y_4) \wedge J \wedge J. \quad (6)$$

as shown in appendix A. All M-theory volumes are expressed in units of l_M . The two-form J is the Kähler form of Y_4 , and $c_2(Y_4)$, $c_3(Y_4)$ denote the second and third Chern class of the tangent bundle of Y_4 , respectively. The quantum volume \mathcal{V}_4 contains an M-theoretic correction of order l_M^6 to the classical volume \mathcal{V}_4 of the internal fourfold Y_4 given by the first term in (5). Both corrections only depend on the Kähler structure of Y_4 and do not introduce mixing with its complex structure. Note that the correction to the 3d Einstein-Hilbert term was already anticipated in [10, 11].

We close this section by noting that the corrections in (5) and (6) are already present in 5d compactifications of M-theory on a Calabi-Yau threefold Y_3 . The classical volume of the threefold receives a correction proportional to the Euler number of the threefold. The four-derivative term in (3) is induced due to a non-trivial integral $\int c_2(Y_3) \wedge J$. This term is the supersymmetric completion of the mixed gauge-gravitational Chern-Simons term $A^\Lambda \wedge \text{Tr}(\mathcal{R}^{(5)} \wedge \mathcal{R}^{(5)})$, where the vectors A^Λ are the supersymmetric partners of the Kähler structure deformations.

III. LIFT TO F-THEORY

We next use the duality between M-theory and F-theory to lift the l_M -corrections in (3) to α' -corrections of the 4d effective theory arising from F-theory compactified on Y_4 . In order to do that, we first require that Y_4 admits an elliptic fibration over a three-dimensional Kähler base B_3 . In this case we can use adjunction formulæ to express Chern classes of Y_4 in terms of Chern classes of B_3 . For simplicity, let us restrict to a smooth Weierstrass model, i.e. a geometry without non-Abelian singularities, that can be embedded in an ambient fibration with typical fibers being the weighted projective space \mathbb{WP}_{231}^2 . This implies having just two types of divisors D_Λ , $\Lambda = 1, \dots, h^{1,1}(Y_4)$. There is the horizontal divisor corresponding to the 0-section D_0 , and the vertical divisors D_α , $\alpha = 1, \dots, h^{1,1}(B_3)$, corresponding to elliptic fibrations over base divisors. Denoting the Poincaré-dual two-forms to the divisors by $\omega_\Lambda = (\omega_0, \omega_\alpha)$, we expand

$$J = v^0 \omega_0 + v^\alpha \omega_\alpha, \quad (7)$$

where v^0 is the volume of the elliptic fiber. Using adjunction formulæ one derives

$$c_3(Y_4) = c_3 - c_1 c_2 - 60 c_1^3 - 60 \omega_0 c_1^2, \quad (8)$$

$$c_2(Y_4) = c_2 + 11 c_1^2 + 12 \omega_0 c_1, \quad (9)$$

where the c_i on the r.h.s. of these expressions denote the Chern classes of B_3 pulled-back to Y_4 .

In order to take the F-theory limit of the expression (3), we need the relation between the 11d Planck length l_M and the string length l_s . Using M/F-theory duality one obtains

$$l_s = l_M (2\pi)^{1/2} (v^0)^{-1/4}, \quad r = (2\pi)^{3/2} (v^0)^{-3/4}. \quad (10)$$

The second equation above is the expression for the radius of the 4d/3d circle in string units. In the F-theory limit one sends $v^0 \rightarrow 0$ and thus decompactifies the fourth dimension by sending $r \rightarrow \infty$. Henceforth, all volumes of the base B_3 will be expressed in units of l_s .

We now have to retain the leading order terms in (3) in the limit of vanishing fiber volume $v^0 \rightarrow 0$. We introduce a small parameter ϵ and express the scaling of the dimensionless fields by writing $v^0 \sim \epsilon$. As explained in [3, 19] one finds $v^\alpha \sim \epsilon^{-1/2}$ and infers the scaling behavior of the classical volume of Y_4 to be $\mathcal{V}_4 \sim \epsilon^{-1/2}$. In the following we use the subscript b to denote quantities of the base that are finite in the limit $\epsilon \rightarrow 0$. In particular, one has

$$2\pi v_b^\alpha = \sqrt{v^0} v^\alpha, \quad (11)$$

which holds in the strict $\epsilon \rightarrow 0$ limit. Note that v_b^α are the volumes of two-cycles of the base in the Einstein frame. Inserting (10) and (11) into (3), and neglecting all terms that vanish for ϵ going to zero, we obtain

$$S_{(4)} \supset \frac{1}{(2\pi)^7 l_s^2} \int *_4 \mathbf{1} \{ \tilde{\mathcal{V}}_3^b R_{sc}^{(4)} + l_s^2 \tilde{\mathcal{V}}_2^b |\mathcal{R}^{(4)}|^2 \}, \quad (12)$$

where

$$\tilde{\mathcal{V}}_3^b = \frac{1}{3!} \int_{B_3} J_b^3 - \frac{5}{8} \int_{B_3} c_1^2(B_3) \wedge J_b, \quad (13)$$

$$\tilde{\mathcal{V}}_2^b = \frac{1}{8} \int_{B_3} c_1(B_3) \wedge J_b^2. \quad (14)$$

Here $\tilde{\mathcal{V}}_3^b$ is now the quantum volume of the base B_3 .

While (14) enters the 4d effective action (12) as a higher derivative correction, (13) contains a correction to the classical volume of the base. Therefore, the α' correction in (13) induces a modified F-theory Kähler potential. Indeed, starting from the M-theory Kähler potential, the F-theory limit gives

$$K^M = -3 \log \tilde{\mathcal{V}}_4 \longrightarrow \log R + K^F, \quad (15)$$

where

$$K^F = \log \left[\left(\frac{1}{3!} L_b^\alpha L_b^\beta L_b^\gamma - \frac{5}{8 \tilde{\mathcal{V}}_2^b} L_b^\alpha k^\beta k^\gamma \right) \mathcal{K}_{\alpha\beta\gamma} \right]. \quad (16)$$

The L variables are the two-cycle volumes normalized by the total quantum volume of the internal space, i.e. we set

$$L^\Lambda = \frac{v^\Lambda}{\mathcal{V}_4}, \quad L^0 \equiv R, \quad L_b^\alpha = \frac{v_b^\alpha}{\mathcal{V}_4^b}. \quad (17)$$

$\mathcal{K}_{\alpha\beta\gamma}$ is the triple intersection matrix of the base and k^α are the expansion coefficients in $c_1(B_3) = k^\alpha \omega_\alpha$. Equation (15) contains the divergent term $\log R$ that is needed in the decompactification of the fourth dimension. The term K^F given in (16) is finite in the F-theory limit and we have dropped all other terms in $\tilde{\mathcal{V}}_4$ that vanish for $\epsilon \rightarrow 0$. Let us stress that K^F has to be evaluated as a function of the complex $\mathcal{N} = 1$ Kähler coordinates as we discuss in section V.

Before giving the Type IIB string interpretation of the α' corrections in (12), let us comment on some special cases. First of all, when the elliptic fibration is trivial, i.e. $Y_4 = X_3 \times T^2$ with X_3 being a Calabi-Yau threefold, then $c_2(Y_4) = c_2(X_3)$ and $c_3(Y_4) = c_3(X_3)$. Since these have no components along the fiber, all corrections in (3) go to zero and the α' corrections in (12) are absent in the resulting $\mathcal{N} = 2$ theory. Another $\mathcal{N} = 2$ corner of F-theory vacua is reached by taking $Y_4 = K3 \times K3$, a configuration studied in [17] with a focus on α' corrections. In this case $c_3(Y_4) = 0$ and the volume correction (13) vanishes identically. In contrast, both corrections are non-vanishing for 6d, $\mathcal{N} = 1$ vacua arising from F-theory on elliptically fibered Calabi-Yau threefolds with classical action derived in [20, 21]. The terms are generated by taking the F-theory limit of the 5d theory briefly discussed at the end of section II. Since the threefold volume is part of the 6d universal hypermultiplet, the volume correction descends to a modification of the hypermultiplet metric of the 4d, $\mathcal{N} = 2$ theory obtained upon further compactification on T^2 . The impact of this correction will, however, crucially depend on the definition of the $\mathcal{N} = 2$ hypermultiplet coordinates. In summary, comparing all these setups one suspects that the volume correction (13) relies on the presence of intersecting seven-branes but its significance changes on backgrounds with different number of supercharges. This will indeed be confirmed by the analysis of section IV.

We close this section with two important remarks. Firstly, we stress that there are several additional l_M -corrections to the fourfold volume surviving the F-theory limit. To see this consider the case without seven-branes having a product geometry $Y_4 = X_3 \times T^2$. In this situation corrections involving the Type IIB axio-dilaton τ have been computed by integrating out the whole tower of T^2 Kaluza-Klein modes of the 11d supergravity multiplet [5]. This gives the following corrections to the fourfold volume

$$\Delta \mathcal{V}_4^{\mathcal{N}=2} \sim \frac{\chi(X_3)}{(v^0)^{1/2}} E_{3/2}(\tau, \bar{\tau}), \quad (18)$$

that depends on τ through the non-holomorphic Eisenstein series $E_{3/2}$ (see also [22]) and has the correct scaling

behavior to survive the F-theory limit.

Our second remark concerns the compactification of the 11d action (1) on $Y_4 \times S^1$, giving rise to Type IIA string theory in two dimensions. The resulting 2d string frame Einstein-Hilbert term takes the form

$$S_{(2)} \supset \frac{1}{(2\pi)^7} \int *_2 \mathbf{1} g_{\text{IIA}}^{-2} \tilde{\mathcal{V}}_4^s R, \quad (19)$$

where we have used that the length of S^1 is $2\pi g_{\text{IIA}}^{2/3} l_M = 2\pi g_{\text{IIA}} l_s$ in terms of the Type IIA string coupling. $\tilde{\mathcal{V}}_4^s$ denotes the quantum volume of the fourfold in units of l_s and it takes the form

$$\tilde{\mathcal{V}}_4^s = \mathcal{V}_4^s - \frac{1}{8} \left(\zeta(3) - g_{\text{IIA}}^2 \frac{\pi^2}{3} \right) \int_{Y_4} c_3(Y_4) \wedge J. \quad (20)$$

The above correction contains two pieces already present in the 10d Type IIA action as R^4 couplings [5]. The first term in the brackets in (20) is tree-level in string perturbation theory and arises from integrating out S^1 Kaluza-Klein modes analogous to the derivation mentioned for (18). The second arises at one-loop of closed strings and is identified with the circle reduction of the volume correction in (3). We stress that the sign difference in the two contributions arises due to their origin in distinct R^4 couplings in 10d [8, 9]. The $\zeta(3)$ -part can also be derived using mirror symmetry or localization techniques as done in [23, 24]. However, it vanishes in the M-theory limit $g_{\text{IIA}} \rightarrow \infty$, and hence is of no relevance for the present purposes.

IV. STRING THEORY INTERPRETATION

In this section we interpret the corrections in (12) in the weak string-coupling limit considered by Sen [13]. This limit is performed in the complex structure moduli space of Y_4 and gives a weakly coupled description of F-theory in terms of Type IIB string theory on a Calabi-Yau threefold X_3 with an O7-plane and D7-branes. The Calabi-Yau threefold is a double cover of the base B_3 branched along the O7-plane. The class of this branching locus is the pull-back of $c_1(B_3)$ to X_3 . When non-Abelian singularities are absent in F-theory, as in the case we consider, the corresponding Sen limit contains a single recombined D7-brane wrapping a divisor of class $8c_1(B_3)$, as required by seven-brane tadpole cancellation. This D7-brane has the characteristic Whitney-umbrella shape [25, 26].

We first discuss the volume correction in (13). For this correction the intersection curve of the D7-brane with the O7-plane plays a crucial role. It is a double curve with additional pinch point singularities. However, all we need in the following is its volume in X_3 given by

$$\mathcal{V}_{D7 \cap O7} = 8 \int_{X_3} c_1^2(B_3) \wedge J_b, \quad (21)$$

where we omitted the pullback map from B_3 to its double cover X_3 in the integrand. Since the intersection numbers of X_3 are twice the ones of B_3 , we can immediately read off from (13) the induced correction to the classical volume of the Calabi-Yau threefold in units of l_s . Hence we find in the ten-dimensional Einstein frame the corrected threefold volume

$$\tilde{\mathcal{V}}_3 = \mathcal{V}_3 - \frac{5}{64} \mathcal{V}_{D7 \cap O7}, \quad (22)$$

where \mathcal{V}_3 is the classical volume of X_3 that is twice the classical volume of B_3 . Note that the quantum correction in (22) can alternatively be expressed in terms of the volume of the self-intersection curve of the O7-plane by using tadpole cancellation. We stress that the correction is of order α'^2 since two of the original six derivatives in M-theory have been absorbed by the integration on the elliptic fiber.

It is worth noting that the non-triviality of the elliptic fibration causes the appearance of an α' correction already at order two, a phenomenon also observed in [17] for a different correction in F-theory compactifications on $K3 \times K3$. Therefore, in the large volume regime, the correction in (22) is, at the level of the Kähler potential, leading with respect to the α'^3 correction of [27] that depends on the Type IIB string coupling and the Euler characteristic of X_3 . The latter is different in nature as it is inherited from an orientable closed string correction of the parent $\mathcal{N} = 2$ theory arising from compactification of Type IIB on X_3 . Indeed it can be easily extracted from the tree-level term of (18), which gives, in units of l_s , $\Delta \mathcal{V}_3^{\mathcal{N}=2} \sim \chi(X_3) \zeta(3) \text{Im} \tau^{3/2}$. It would be very interesting to compute it directly in a full-fledged F-theory compactification with 4d, $\mathcal{N} = 1$ supersymmetry.

In order to give the string theory interpretation of the correction in (22) we have to identify the string amplitude capturing it. We first look at the 4d effective action in the string frame with Einstein-Hilbert term

$$S_{(4)} \supset \frac{1}{(2\pi)^7 l_s^2} \int *_{\mathbf{4}} \mathbf{1} \left\{ \frac{\mathcal{V}_3^s}{g_{\text{IIB}}^2} - \frac{5\mathcal{V}_{D7 \cap O7}^s}{64 g_{\text{IIB}}} \right\} R_{\text{sc}}^s, \quad (23)$$

where the superscript s denotes quantities computed using the string frame metric. Let us indicate which string amplitude might generate the correction in (23). Recall that the power of the string coupling constant in a given amplitude coincides with $-\chi(\Sigma)$ modified by the number of insertions of vertex operators on the string world-sheet Σ . Both contributions in (23) are expected to arise from amplitudes with two graviton insertions and we study the relative g_s -power of the two terms. The general formula for the Euler number of Riemann surfaces, possibly non-orientable and with boundaries, is

$$\chi(\Sigma) = 2 - 2g - b - c, \quad (24)$$

where g, b, c denote the genus, the number of boundaries, and the number of cross caps, respectively. Therefore, we immediately see that the volume correction in (22)

should arise from a string amplitude that involves the sum over two topologies: The disk ($g = c = 0, b = 1$) and the projective plane ($g = b = 0, c = 1$). They correspond to the tree-level of orientable open strings and non-orientable closed strings, respectively. This interpretation seemingly contradicts the expectation that such a correction arises at open-string one-loop level [14]. We hope to clarify this point in future work by performing a direct string computation. Let us stress that we cannot exclude the possibility that there are further corrections in a fully backreacted M-theory reduction lifted to F-theory that have the same α' -order and field dependence as the volume correction found here.

Let us next give a string theory interpretation of the 4d higher derivative correction in (12). In fact, at weak string coupling, the coefficient (14) can be written as

$$\tilde{\mathcal{V}}_2^{\text{IIB}} = \frac{1}{96} (\mathcal{V}_{D7} + 4\mathcal{V}_{O7}), \quad (25)$$

where \mathcal{V}_{D7} and \mathcal{V}_{O7} are the volumes of the D7-brane and the O7-plane in X_3 , respectively. Both volumes are in the Einstein frame and in units of l_s . By tadpole cancellation one has $\mathcal{V}_{D7} = 8\mathcal{V}_{O7}$. However, in (25) we have split the volumes according to the appearance of the corresponding divisors in the F-theory discriminant. The relative factor in the volume split is in agreement with the relative factor in the higher curvature terms of the Chern-Simons actions of D7-branes and O7-planes. These have been studied to derive the 4d higher curvature term proportional to $\text{Tr}(\mathcal{R}^{(4)} \wedge \mathcal{R}^{(4)})$ in [28], which is the supersymmetric partner of the $\text{Tr}(\mathcal{R}^{(4)} \wedge *_4 \mathcal{R}^{(4)})$ term in (12). Translated to the string frame the higher derivative correction in (12) becomes

$$S_{(4)} \supset \frac{1}{96 (2\pi)^7} \int *_{\mathbf{4}} \mathbf{1} \frac{(\mathcal{V}_{D7}^s + 4\mathcal{V}_{O7}^s)}{g_{\text{IIB}}} |\mathcal{R}_s^{(4)}|^2, \quad (26)$$

where we see that this correction has the same string loop order as the one in (23). This term is expected to directly arise from a higher curvature correction of the string-tree-level Dirac-Born-Infeld action on the D7-brane and O7-plane as discussed in [15] (see, in particular, equation (3.5)).

V. REMARKS ON THE KÄHLER POTENTIAL AND TYPE IIB VACUA

In this final section we comment on the structure of the corrected 4d, $\mathcal{N} = 1$ Kähler potential K^F given in (16) and the vacua induced by this modification. In order to derive the kinetic terms of the moduli and the scalar potential, one first needs to express K^F in terms of the $h^{1,1}(B_3)$ correct $\mathcal{N} = 1$ complex coordinates T_α . Starting in M-theory one has $h^{1,1}(Y_4)$ complex coordinates T_Λ . Classically the real parts of T_Λ are the volumes of the divisors of Y_4 , while the imaginary part is the integral of the M-theory six-form over the same divisors. Including

higher curvature corrections also the T_Λ might be shifted as

$$\text{Re } T_\Lambda = \frac{1}{3!} \int_{Y_4} J^3 \wedge \omega_\Lambda + \kappa \chi(D_\Lambda), \quad (27)$$

where $\chi(D_\Lambda) = \int_{Y_4} c_3(Y_4) \wedge \omega_\Lambda$, κ is some constant, and volumes are expressed in units of l_M . Such a shift could be expected in analogy to the proposal of [3] that the $\text{Re } T_\Lambda$ are related to the periods of the mirror Calabi-Yau fourfold. To derive corrections to the T_Λ the modification of their kinetic terms has to be computed. For the correction considered here one would have to dimensionally reduce the 11d higher curvature term in (1) including fluctuations of the Calabi-Yau metric. However, since the correction in (27) is simply a constant shift it will not affect the derivation of the Kähler metric or the scalar potential. Therefore, we will set $\kappa = 0$ in the following discussion.

To evaluate the Kähler potential in terms of the T_Λ , we first recall the definition of the L^Λ in (17) and note that

$$\mathcal{V}_4^{-3} = \tilde{\mathcal{V}}_4^{-3} \left(1 - \lambda \chi(D_\Lambda) L^\Lambda \right), \quad (28)$$

which can be easily derived from (5) setting $\lambda = \frac{\pi^2}{24}$ and $\mathcal{V}_4 = \frac{1}{4!} \int J^4$. One then shows that the no-scale-like property of the corrected M-theory Kähler potential is broken as

$$K_{T_\Lambda} K^{T_\Lambda \bar{T}_\Sigma} K_{\bar{T}_\Sigma} = 4 - \frac{\lambda \chi(D_\Lambda) L^\Lambda}{1 - \lambda \chi(D_\Lambda) L^\Lambda}, \quad (29)$$

where $K^{T_\Lambda \bar{T}_\Sigma}$ is the inverse Kähler metric. In order to express the M-theory Kähler potential in terms of the T_Λ -moduli, we solve (27) for L^Λ , and insert the solution back into (15). Strictly speaking one would next have to perform a Legendre transformation replacing $\text{Re } T_0$ with R and work with the modified kinetic potential $\tilde{K}(R, T_\alpha)$. However, in the F-theory limit one finds the expression (16) that we will use in the following. The correction in K^F also breaks the no-scale property as

$$K_{T_\alpha}^F K^{F T_\alpha \bar{T}_\alpha} K_{\bar{T}_\alpha}^F = 3 - \frac{\chi_\alpha L_b^\alpha}{96 - \chi_\alpha L_b^\alpha}, \quad (30)$$

where $\chi_\alpha = \chi(D_\alpha) = -60 \int_{B_3} c_1^2(B_3) \wedge \omega_\alpha$, and all quantities on the r.h.s. are quantities of the base B_3 . Analytically expressing K^F in terms of T_α can be straightforwardly done in a toy model with a single T_α -modulus, which we denote by T . In this case one finds

$$\begin{aligned} K^F &= -2 \log \tilde{\mathcal{V}}_3 \\ &= -\log \left[\text{Re } T \left(\text{Re } T - \frac{15}{8} k^2 \right)^2 \right], \end{aligned} \quad (31)$$

with k the first Chern number of B_3 .

The F-theory Kähler potential (31) can be expanded in powers of $\text{Re } T^{-1}$ in the large volume regime and it

can be seen that the leading correction to the classical Kähler potential is an homogeneous expression of degree -2 in the 2-cycle volumes of the base. According to [29–31], this implies that it vanishes in the scalar potential. Nevertheless, higher order corrections persist also at the level of the scalar potential and it would be interesting to work out their consequences on Kähler moduli stabilization within the large volume scenario [32, 33]. We hope to come back to this in a future work.

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Appendix A: Computation

We use the conventions of [34] for the definition of the Riemann tensor and related quantities. The background geometry is the product manifold $M_{11} = \mathbb{R}^{1,2} \times Y_4$, where the flat space has signature $\{-, +, +\}$ and Y_4 is the Calabi-Yau fourfold. External indices are denoted by μ, μ' . For the coordinates on Y_4 we use real and complex indices denoted by a, a' and $\alpha, \beta, \gamma, \delta$, respectively. Indices of the coordinates of the total space M_{11} will be written in capital Latin letters N, N' . Furthermore, the convention for the totally anti-symmetric tensor in Lorentzian space is $\epsilon_{012\dots 10} = \epsilon_{012} = +1$.

The curvature two-form for Hermitian manifolds is defined as

$$\mathcal{R}^\alpha{}_\beta = R^\alpha{}_{\beta\gamma\bar{\delta}} dz^\gamma \wedge d\bar{z}^{\bar{\delta}}, \quad (A1)$$

and one has

$$\text{Tr } \mathcal{R} = R^\alpha{}_{\alpha\gamma\bar{\delta}} dz^\gamma \wedge d\bar{z}^{\bar{\delta}}, \quad (A2)$$

$$\text{Tr } \mathcal{R}^2 = R^\alpha{}_{\beta\gamma\bar{\delta}} R^\beta{}_{\alpha\gamma_1\bar{\delta}_1} dz^\gamma \wedge d\bar{z}^{\bar{\delta}} \wedge dz^{\gamma_1} \wedge d\bar{z}^{\bar{\delta}_1},$$

$$\text{Tr } \mathcal{R}^3 = R^\alpha{}_{\beta\gamma\bar{\delta}} R^\beta{}_{\beta_1\gamma_1\bar{\delta}_1} R^{\beta_1}{}_{\alpha\gamma_2\bar{\delta}_2} dz^\gamma \wedge d\bar{z}^{\bar{\delta}} \dots d\bar{z}^{\bar{\delta}_2}.$$

The correction to the 11d Einstein-Hilbert term in (1) is given by \mathcal{J}_0 schematically defined in (2) to be

$$\mathcal{J}_0 = t_8 t_8 R^4 - \frac{1}{4!} \epsilon_{11} \epsilon_{11} R^4. \quad (A3)$$

Following [35] we define in real coordinates

$$\begin{aligned} t_8^{a_1 a_2 \dots a_8} &= -2 \left(\delta^{[a_1 [a_3 | \delta^{[a_2 | a_4]} \delta^{[a_5 \langle a_7 | \delta^{[a_6 | a_8]} \right.} \\ &\quad + \delta^{[a_1 [a_5 | \delta^{[a_2 | a_6]} \delta^{[a_3 \langle a_7 | \delta^{[a_4 | a_8]} \right.} \\ &\quad + \delta^{[a_1 [a_7 | \delta^{[a_2 | a_8]} \delta^{[a_3 \langle a_5 | \delta^{[a_4 | a_6]} \rangle] } \\ &\quad + 8 \left(\delta^{\langle a_2 | [a_3 \delta^{a_4]} [a_5 \delta^{a_6}] [a_7 \delta^{a_8}] | a_1 \rangle} \right. \\ &\quad + \delta^{\langle a_2 | [a_5 \delta^{a_6}] [a_3 \delta^{a_4}] [a_7 \delta^{a_8}] | a_1 \rangle} \\ &\quad \left. \left. + \delta^{\langle a_2 | [a_5 \delta^{a_6}] [a_7 \delta^{a_8}] [a_3 \delta^{a_4}] | a_1 \rangle \right) \right). \end{aligned} \quad (A4)$$

The symbols $[\]$, $[\]$, $[\]$, $\langle \ \rangle$ denote anti-symmetrization and indices in between two vertical lines $|$ are omitted in the respective bracket. The expression is anti-symmetrized in the following pairs of indices $(a_1 a_2)$, $(a_3 a_4)$, $(a_5 a_6)$, $(a_7 a_8)$ respectively. Thus we have for the term $t_8 t_8 R^4$ in real coordinates

$$t_8 t_8 R^4 = t_{8a_1 \dots a_8} t_8^{a'_1 \dots a'_8} R^{a_1 a_2}_{a'_1 a'_2} \dots R^{a_7 a_8}_{a'_7 a'_8}. \quad (\text{A5})$$

The second term in (A3) can be written as

$$\frac{1}{4!} \epsilon_{11} \epsilon_{11} R^4 = \frac{1}{4} E_8(M_{11}), \quad (\text{A6})$$

where one uses the general definition in real coordinates

$$E_n(M_D) = \frac{1}{(D-n)!} \epsilon_{N_1 \dots N_n} \epsilon^{N_1 \dots N_{D-n} N'_{D-n+1} \dots N'_n} R^{N_{D-n+1} N_{D-n+2}}_{N'_{D-n+1} N'_{D-n+2}} \dots R^{N_{D-1} N_D}_{N'_{D-1} N'_D}, \quad (\text{A7})$$

where $n > 0$ and D being the real dimension of the manifold M_D .

The Chern classes can be expressed in terms of the curvature two-form \mathcal{R} as

$$\begin{aligned} c_1 &= i \text{Tr } \mathcal{R}, \quad c_2 = \frac{1}{2!} (\text{Tr } \mathcal{R}^2 - (\text{Tr } \mathcal{R})^2), \quad (\text{A8}) \\ c_3 &= \frac{1}{3} c_1 c_2 + \frac{1}{3} c_1 \wedge \text{Tr } \mathcal{R}^2 - \frac{i}{3} \text{Tr } \mathcal{R}^3 \\ c_4 &= \frac{1}{24} (c_1^4 - 6 c_1^2 \text{Tr } \mathcal{R}^2 - 8 i c_1 \text{Tr } \mathcal{R}^3) \\ &\quad + \frac{1}{8} ((\text{Tr } \mathcal{R}^2)^2 - 2 \text{Tr } \mathcal{R}^4). \end{aligned}$$

The Chern classes of a Calabi-Yau fourfold reduce to $c_3(Y_4) = -\frac{i}{3} \text{Tr } \mathcal{R}^3$ and $c_4 = \frac{1}{8} ((\text{Tr } \mathcal{R}^2)^2 - 2 \text{Tr } \mathcal{R}^4)$.

Let us first compute $E_8(M_3 \times M_8)$ for a generic product space. By using the definition (A7), splitting indices and applying Schouten identities it is straightforward to show that

$$E_8(M_3 \times M_8) = -E_8(M_8) + 4 E_2(M_3) E_6(M_8), \quad (\text{A9})$$

where $E_2(M_3) = -2R_{\text{sc}}^{(3)}$ and

$$E_6(M_8) = 6! R^{[a_1 a_2}_{a_1 a_2} \dots R^{a_5 a_6]}_{a_5 a_6}. \quad (\text{A10})$$

Next we evaluate $c_3 \wedge J$ on Y_4 , where $J = i g_{\alpha\bar{\beta}} dz^\alpha \wedge d\bar{z}^{\bar{\beta}}$ is the Kähler form. Expressing c_3 in holomorphic coordinates we find

$$c_3 \wedge J = 2 R^{\alpha_0}_{\beta_0 \gamma_0} R^{[\beta_0]}_{\beta_1 \gamma_1} R^{\gamma_1}_{\beta_1 \gamma_1} R^{[\beta_1]}_{\alpha_0 \gamma_2} R^{\gamma_2]}_{\alpha_0 \gamma_2} *_{\mathbf{8}} \mathbf{1}. \quad (\text{A11})$$

In order to compare (A10) and (A11) one has to change from real coordinates to complex ones. For $M_8 = Y_4$ one finds

$$E_6(Y_4) *_{\mathbf{8}} \mathbf{1} = 128 \left(R^{\alpha_0}_{\beta_0 \gamma_0} R^{\delta_0}_{\alpha_1 \delta_0} R^{\delta_1}_{\alpha_0 \delta_1} R^{\alpha_1}_{\alpha_0 \delta_1} \gamma_0 \right. \quad (\text{A12})$$

$\left. + R^{\alpha_0}_{\beta_0 \gamma_0} R^{\delta_0}_{\alpha_1 \delta_1} R^{\gamma_0}_{\alpha_1 \delta_0} R^{\alpha_1}_{\alpha_0 \delta_0} \delta_1 \right) *_{\mathbf{8}} \mathbf{1}.$
Comparing (A11) and (A12) we find

$$E_6(Y_4) *_{\mathbf{8}} \mathbf{1} = 3 \cdot 2^7 c_3 \wedge J. \quad (\text{A13})$$

Additionally we have

$$\frac{1}{4} E_8(Y_4) *_{\mathbf{8}} \mathbf{1} = 1536 c_4. \quad (\text{A14})$$

Finally we find for the reduction of $t_8 t_8 R^4$ the terms

$$t_8 t_8 R^4 *_{\mathbf{8}} \mathbf{1} = -96 (R^\mu_\nu)_{\mu' \nu'} (R^\nu_\mu)^{\mu' \nu'} (R^\alpha_\beta)_{\gamma_0}^{\delta_1} (R^\beta_\alpha)_{\delta_1}^{\gamma_0} *_{\mathbf{8}} \mathbf{1} \quad (\text{A15})$$

where the dots indicate purely external terms. To rewrite this result let us consider the following terms

$$\begin{aligned} c_2 \wedge J^2 &= -2 \delta^{[\gamma_0}_{\delta_0} \delta^{\gamma_1]}_{\delta_1} (R^\alpha_\beta)_{\gamma_0}^{\delta_0} (R^\beta_\alpha)_{\gamma_1}^{\delta_1} *_{\mathbf{8}} \mathbf{1} \\ &= -(R^\alpha_\beta)_{\gamma_0}^{\delta_1} (R^\beta_\alpha)_{\delta_1}^{\gamma_0} *_{\mathbf{8}} \mathbf{1}. \end{aligned} \quad (\text{A16})$$

Furthermore, we find

$$\text{Tr} [\mathcal{R}^{(3)} \wedge *_3 \mathcal{R}^{(3)}] = \frac{1}{8} (R^\mu_\nu)_{\mu' \nu'} (R^\nu_\mu)^{\mu' \nu'} *_3 \mathbf{1}, \quad (\text{A17})$$

with $\mathcal{R}^{(3)}$ being the real curvature two-form as in (4). Hence we conclude

$$t_8 t_8 R^4 *_{11} \mathbf{1} = 3 \cdot 2^8 \text{Tr} [\mathcal{R}^{(3)} \wedge *_3 \mathcal{R}^{(3)}] \wedge (c_2 \wedge J^2) \quad (\text{A18})$$

where we have omitted the same purely external terms as in (A15).

To conclude we use (A3), (A9), (A14) and (A18) to find for the internal terms of \mathcal{J}_0

$$(\mathcal{J}_0)_{\text{int}} *_{\mathbf{8}} \mathbf{1} = \left[(t_8 t_8 R^4)_{\text{int}} + \frac{1}{4} E_8(Y_4) \right] *_{\mathbf{8}} \mathbf{1} = 3072 c_4, \quad (\text{A19})$$

which integrate to 3072χ on Y_4 . The linear combination with a different relative sign in equation (A19) obviously vanishes on Y_4 . This is of physical importance as discussed e.g. in [36].

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